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ON FORECASTING WINTER PRECIPITATION AMOUNTS AT WASHINGTON, D. C.

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ABSTRACT

A simplified model of the rainfall mechanism is used as a guide in developing a statistical formula, with readily measured variables as arguments, for forecasting precipitation amounts in winter at Washington, D. C. The skill of the forecasting method is tested on independent data and for snow cases. To simplify computations involved in using the formula, a nomograph is devised. Suggestions are given for further work on the method.

INTRODUCTION

This study is aimed at developing a logical method of forecasting the amount of precipitation that will fall at Washington, D. C. The technique employed is to use a simplified model of the rainfall mechanism as a guide in developing a statistical forecasting formula which uses readily measured variables as arguments. Only the winter season is treated and the forecast is for the period 0800 to 2000 EST, the forecast period usually termed "today". The data used in making the forecast are restricted to those available to the forecaster at 0130 EST; the precipitation amount referred to is that recorded in inches by the rain gage at Washington National Airport during the period for which the forecast is made.

THEORETICAL BACKGROUND

The exact mechanism by which moisture is condensed and precipitated from the atmosphere is not completely understood. Fulks [1] analyzed a situation in which it was assumed that all condensed water was precipitated and that the vertical velocity and the vertical distribution of moisture were known. The derivation is admittedly an over-simplified one which ignores the problems of the variation in intensity of precipitation and the controversial subject of colloidal instability, as well as the problem of evaporation. Nevertheless, Fulks' work provides a starting point in devising a systematic method for forecasting precipitation. Holmboe, Forsythe, and Gustin [2] present an excellent re-analysis of Fulks' work. They set forth the following formula for computing the rate of precipitation in millimeters per hour:

$$\textit{Precipitation rate} = \sum_{H_1}^{H_2} WP\frac{\Delta H}{100}$$

W is the average vertical velocity through a saturated layer of air.

P is the rate of precipitation from a saturated layer of air ascending with a unit of vertical velocity and is essentially a function of the pressure and mixing ratio.

ΔH is the thickness of a sublayer of saturated air.

H_1 and H_2 are the lower and upper heights respectively, of the layer over which the quantity $WP\frac{\Delta H}{100}$ is summed.

Since only the form of this equation is considered in this study, and no computations are made, it is not necessary to enter into any discussion of units.

The form of this equation indicates that the precipitation is a function of the amount of moisture available and the vertical velocity of the air. Fletcher [3] and Showalter [4] have suggested formulae similar to this in studies of precipitation amounts. Realistic computations of the concurrent vertical velocity are not feasible, and it would be impractical to compute, from theory now available, a forecast of the vertical velocity. The problem of making a forecast of the distribution of moisture and the layers where the air will be saturated is fully as difficult as the problem of forecasting vertical velocity. In order to circumvent these theoretical difficulties the statistical approach may be used; that is, variables which should be related to P and W may be measured from weather charts and correlated with precipitation amounts in the manner suggested by the formula.

TYPING SYSTEM

In order to reduce the number of cases to be handled in the statistical analysis of the data, a typing system was devised. The first factor to be considered, following Brier [5], was the occurrence or nonoccurrence of precipitation upstream from Washington. This denotes the presence or absence of a precipitation mechanism in a position from which it is likely to move toward Washington. On the original sample of data selected for study, there were 113 cases in which the 700-mb. contour through Washington did not cross over a rain area on the surface map between a point 2° of latitude upstream from Washington and 95° west longitude. Of these 113 cases, 108 did not result in precipitation at Washington during the specified forecast period. The heaviest amount among the 5 cases which did result in precipitation showed east-

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erly flow at the surface at Washington and southerly flow aloft, a condition that Brier found very useful in predicting Washington winter precipitation. The data were therefore divided into two types by the following criteria.

Precipitation Type.—The upwind extension of the 700 mb. contour line through Washington crosses an active precipitation area on the 0130 EST surface map between a point 2° upwind and 95° west longitude, or a combination of an easterly surface wind with a southerly 700 mb. wind is found at Washington.

No-precipitation Type.—Neither of the above conditions is met.

In the No-precipitation Type only 3.6 percent of the cases resulted in precipitation during the forecast period and therefore this type was not further considered. The Precipitation Type was characterized by resultant precipitation in 45.5 percent of the cases and is the type for which a statistical formula was developed.

VARIABLES

The next step was the selection of readily measured variables that are related to P and W in Fulks' formula. P is essentially a function of pressure and mixing ratio [2]; it is a single valued function of the mixing ratio at constant pressure. If it is assumed that the mixing ratio at some constant pressure surface is a good index of the moisture in the saturated layer, and if a scheme could be devised to locate a point where this mixing ratio would represent the air crossing a station during a given period, the mixing ratio could be combined with a measure of the vertical velocity to yield an estimate of the precipitation to be expected.

In order to estimate this mixing ratio, a variable Q was defined by the following procedure. The geostrophic wind at the 700 mb. surface was computed at Washington. A distance equal to 12 hours' travel at this speed was measured along the upstream extension of the 700-mb. contour line through Washington. If there was obviously a large change in gradient upstream from Washington, the 12 hours' travel was computed in two steps using 6 hours' travel distance from Washington to a point, and then measuring 6 hours' travel distance farther upstream at the rate of the current geostrophic wind computed at that point. The mixing ratio at 850 mb. was read under the upwind terminus of the 700-mb. 12-hour geostrophic travel; this mixing ratio is the variable Q. The process of moving upstream a distance equal to 12 hours' travel and then using the mixing ratio under that point gives an approximation of the three-dimensional trajectory of the air. The variable Q is directly proportional to P, since it is a mixing ratio measured in a constant pressure surface.

In a paper on vertical velocity Miller [6] shows that there is a correlation between the southerly horizontal wind component and the vertical motion. As a measure of the vertical motion to be expected at Washington the variable G was devised. G is defined by $\frac{Z_{CHS}-Z_{\min}}{D}$ where Z_{CHS} is the height of the 850 mb. surface over Charleston, S. C., $Z_{\min}$ is the lowest height of the 850 mb. surface along a line between Charleston and Little Rock, Ark., except that when Charleston reports the lowest pressure along the line, the height at Little Rock is used as $Z_{\min}$. D is the distance from Charleston to the point of $Z_{\min}$. G is expressed in tens of feet per degree of latitude, and as a measure of southerly wind component should be related to the vertical velocity in such a manner that large positive values of G correspond to large upward velocities.

Another variable, useful in estimating the expected vertical velocity, was considered advisable. In a study of Washington winter precipitation Brier [5] used a variable that he denoted by L. This variable is the lowest latitude reached by the upwind extension of the 700-mb. contour line through Washington between WA and 95° west longitude, except that if WA is at the point of lowest latitude, L is defined as the latitude of the intersection of the contour line through Washington and 95° west longitude. It is measured in degrees of latitude from the equator. The success of this variable is, at least in part, due to the fact that it measures the southerly component of the wind to be expected at Washington, and hence is a measure of the vertical velocity. Since the southerly component will be greatest when the latitude is lowest, L should be related to the vertical velocity in such a manner that small values of L correspond to large upward velocities.

There is no variable in this study to measure the thickness of the saturated layer, ΔH. This may reduce the effectiveness of the system; however, since the variables are approximations, it is felt that the lack of such a measure will not materially affect the results.

FORM OF RELATIONSHIP

Two variables, $1/L$ and G, have been defined which should be proportional to the vertical velocity over Washington during the forecast period. The assumption that these should be combined as a weighted mean seems to be most consistent as an approximation of W which is defined as the mean vertical velocity of the saturated layer. From Fulks' formula it can be seen that Q should be multiplied by the vertical velocity term. Thus, considering the forecast period as a unit of time,

$$\textit{Precipitation} \propto Q(1/L+G)$$

Realizing that the vertical velocity components should probably not be equally weighted, and that constants of conversion and proportionality are involved, the formula,

$$\textit{Precipitation}=a+b\ Q/L+cQG$$

was fitted to the data for the two winters 1944–45 and 1945–46. The mathematical fit by the least squares method yielded the following values for a, b, and c:

$a=-.066$ inches
$b=1.57$ inches$\times$degrees latitude
$c=.0055$ inches$\times$degrees latitude/10's of feet.

These constants give a forecast in inches of precipitation for the period. When the results of applying this formula to the Precipitation Type of the original data were broken down into categorical forecasts, it was found that it showed little skill in separating the rain amounts within categories. See table 1.

A plot of the values of Q/L against QG (fig. 1) with precipitation values entered in the body of the chart indicates that a better job may be done on the categorical forecasts.

Whereas the least square fit reduces the square of the errors to a minimum, it is not necessarily the best forecast equation. It is desirable to minimize the error, but for large errors, it makes little difference if the error is .50 in. or .60 in., and large errors are heavily weighted by least squares. On the other hand, it is probably better to forecast a great number of no-precipitation cases and accept a few light precipitation cases as errors, than to forecast a large number of small amounts and accept

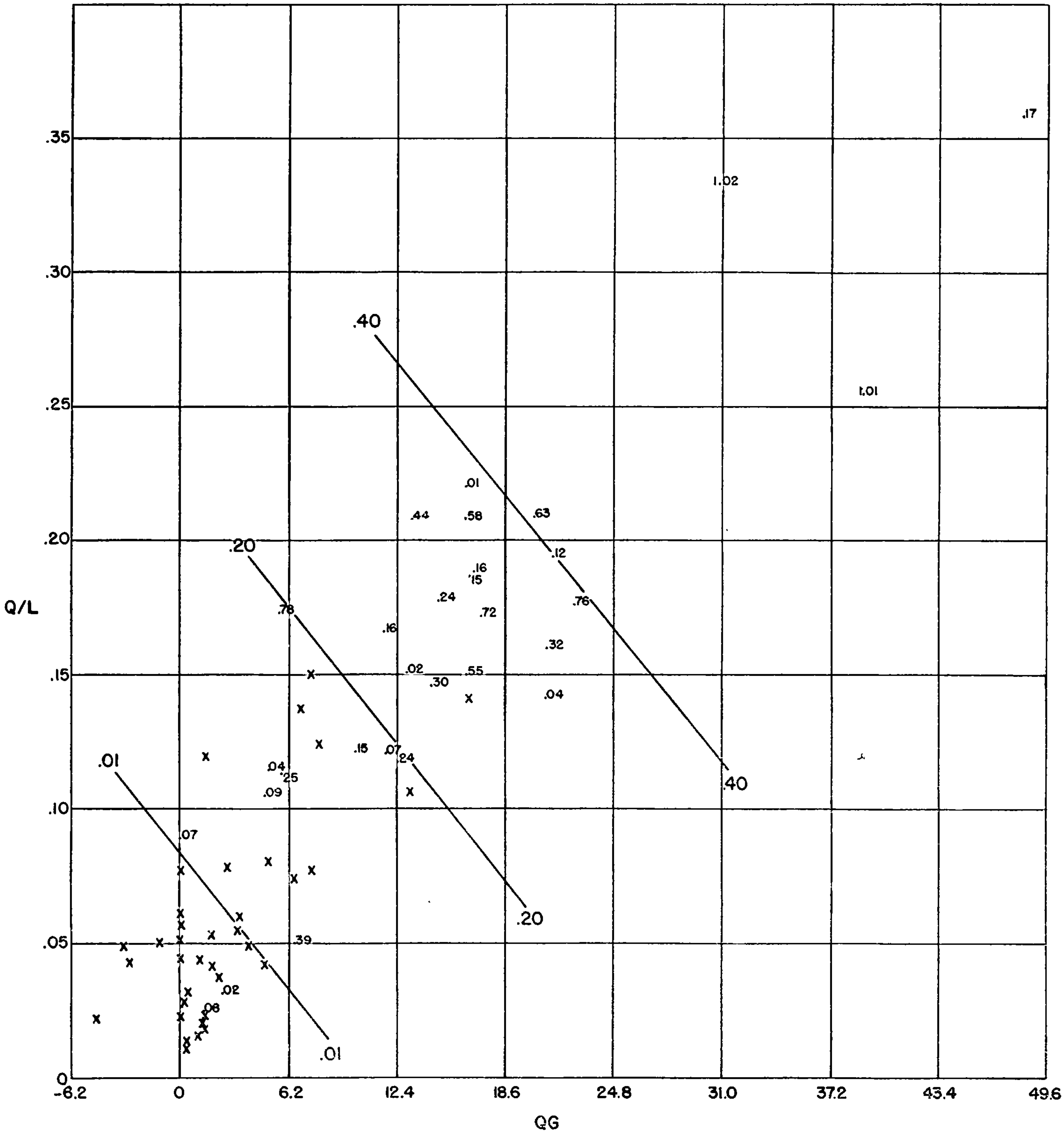

FIGURE 1.—A plot of Q/L against QG with precipitation amounts entered at each point. No measurable precipitation is indicated by a cross. The lines represent the subjectively fitted plane.

many zeros as errors. Therefore a second forecast formula was derived subjectively by fitting a plane to the data in figure 1. This plane was fitted by first drawing the line which seemed to separate the large majority of the cases of no measurable precipitation from the rest of the data and then drawing a .30-in. line parallel to it through an area where the average precipitation was .30 in. Using three points taken from these two lines the following formula was fitted:

$$Precipitation = 1.35\ Q/L + .0111\ QG - .10$$

The constants have the same dimensions as for the mathematically fitted equation.

RESULTS OF TESTS

In addition to the data for the winters of 1943–44 and 1944–45 which were used to derive the typing system and to evaluate the constants in the formulae, the method was tested on the data for the winter of 1945–46. The most convenient way to present the results of these tests is by contingency tables which indicate how well the method discrminates among the various classes of precipitation amounts. The original data were used to determine the class intervals or categories. The first category includes all those cases where no measurable precipitation fell during the verification period. The remaining categories divide the observed precipitation amounts into groups so that there are at least 10 cases in each group and the groups contain approximately the same number of cases. The purpose of this system of classification was to obtain a large enough sample in each category to estimate the skill of the method.

A comparison of tables 1 and 2 (a) brings out the improvement in the categorical forecasts due to the subjectively fitted equation. Tables 2 (a) and 2 (b) show that the formula fits the Precipitation Type on the test data almost as well as it does on the original. Table 3 shows the distribution of precipitation amounts which occurred in the No-precipitation Type cases of the test data. It indicates that the stratification developed from the original data does almost as well on the test data. Tables 4 (a) and 4 (b) show what might be achieved by daily use of the method. For the purpose of this comparison all the No-precipitation Type cases were forecast to give no precipitation. Tables 5 (a) and 5 (b) indicate the skill of the method in discriminating between precipitation and no-precipitation cases.

TABLE 1.—*Four category contingency tables showing the results of applying the mathematically fitted equation to the Precipitation Type cases of the original data: the winters of 1943–44 and 1944–45*

OBSERVED \ FORECAST	0	0.01–.10	0.11–.30	>0.30	Total
0	15	15	6	0	36
0.01–.10	2	1	5	1	9
0.11–.30	0	0	6	4	10
>0.30	0	1	4	6	11
Total	17	17	21	11	66

Percent correct=42
Skill score=.22

TABLE 2.—*Contingency tables showing the results of applying the subjectively fitted formula to the Precipitation Type cases*

(*a*) *Original data: the winters of 1943–44 and 1944–45*

OBSERVED \ FORECAST	0	0.01–.10	0.11–.30	>0.30	Total
0	22	9	5	0	36
0.01–.10	2	2	3	2	9
0.11–.30	0	0	7	3	10
>0.30	0	1	2	8	11
Total	24	12	17	13	66

Percent correct=59
Skill score=.42

(*b*) *Test data: the winter of 1945–46*

OBSERVED \ FORECAST	0	0.01–.10	0.11–.30	>0.30	Total
0	10	3	2	2	17
0.01–.10	1	4	4	2	11
0.11–.30	0	0	3	3	6
>0.30	0	1	0	4	5
Total	11	8	9	11	39

Percent correct=54
Skill score=.39

TABLE 3.—*The distribution of precipitation amounts in the No-Precipitation Type cases of the test data: the winter of 1945–46*

Categories	0	0.01–.10	0.11–.30	Total
Number of cases observed	47	1	3	51

Percentage of cases with precipitation=7.8

TABLE 4.—*Contingency tables showing the results of applying the forecasting method to the entire sample*

(*a*) *Original data: the winters of 1943–44 and 1944–45*

OBSERVED \ FORECAST	0	0.01–.10	0.11–.30	>0.30	Total
0	130	9	5	0	144
0.01–.10	4	2	3	2	11
0.11–.30	2	0	7	3	12
>0.30	0	1	2	8	11
Total	136	12	17	13	178

Percent correct=82
Skill score=.52

(*b*) *Test data: the winter of 1945–46*

OBSERVED \ FORECAST	0	0.01–.10	0.11–.30	>0.30	Total
0	57	3	2	2	64
0.01–.10	2	4	4	2	12
0.11–.30	3	0	3	3	9
>0.30	0	1	0	4	5
Total	62	8	9	11	90

Percent correct=75
Skill score=.49

TABLE 5.—*Two category contingency tables showing the results of using the forecasting method to discriminate between precipitation and no precipitation*

(*a*) *Original data: the winters of 1943–44 and 1944–45*

		FORECAST No Precip.	Precip.	Total	
OBSERVED	No Precip.	130	14	144	Percent correct=89
	Precip.	6	28	34	Skill score=.67
	Total	136	42	178	

(*b*) *Test data: the winter of 1945–46*

		FORECAST No Precip.	Precip.	Total	
OBSERVED	No Precip.	57	7	64	Percent correct=87
	Precip.	5	21	26	Skill score=.68
	Total	62	28	90	

APPLICATION TO SNOW CASES

Because of the great economic importance of heavy snow it is of interest to examine the results of this formula as applied to the cases of snow at Washington. Because snow is less frequent than rain, both samples of data were combined and all the snow cases were collected. When the computed amount of precipitation was correlated with the observed amounts for the twelve snow cases a correlation coefficient of .84 was obtained. Table 6 is a catalogue of these cases. This indicates that the formula does as well forecasting the precipitation amounts for snow as it does the amounts of rain.

TABLE 6.—*A comparison of the observed and computed precipitation amounts for the 12 cases, wherein the precipitation fell as snow in the 3 winters 1943–44, 1944–45, and 1945–46*

Precipitation amounts (inches)

Observed	0.16	0.02	0.03	0.01	0.30	0.07	0.29	0.05	0.03	0.01	0.44
Forecast	.25	0	0	0	.26	.02	.11	.08	.06	.04	.33

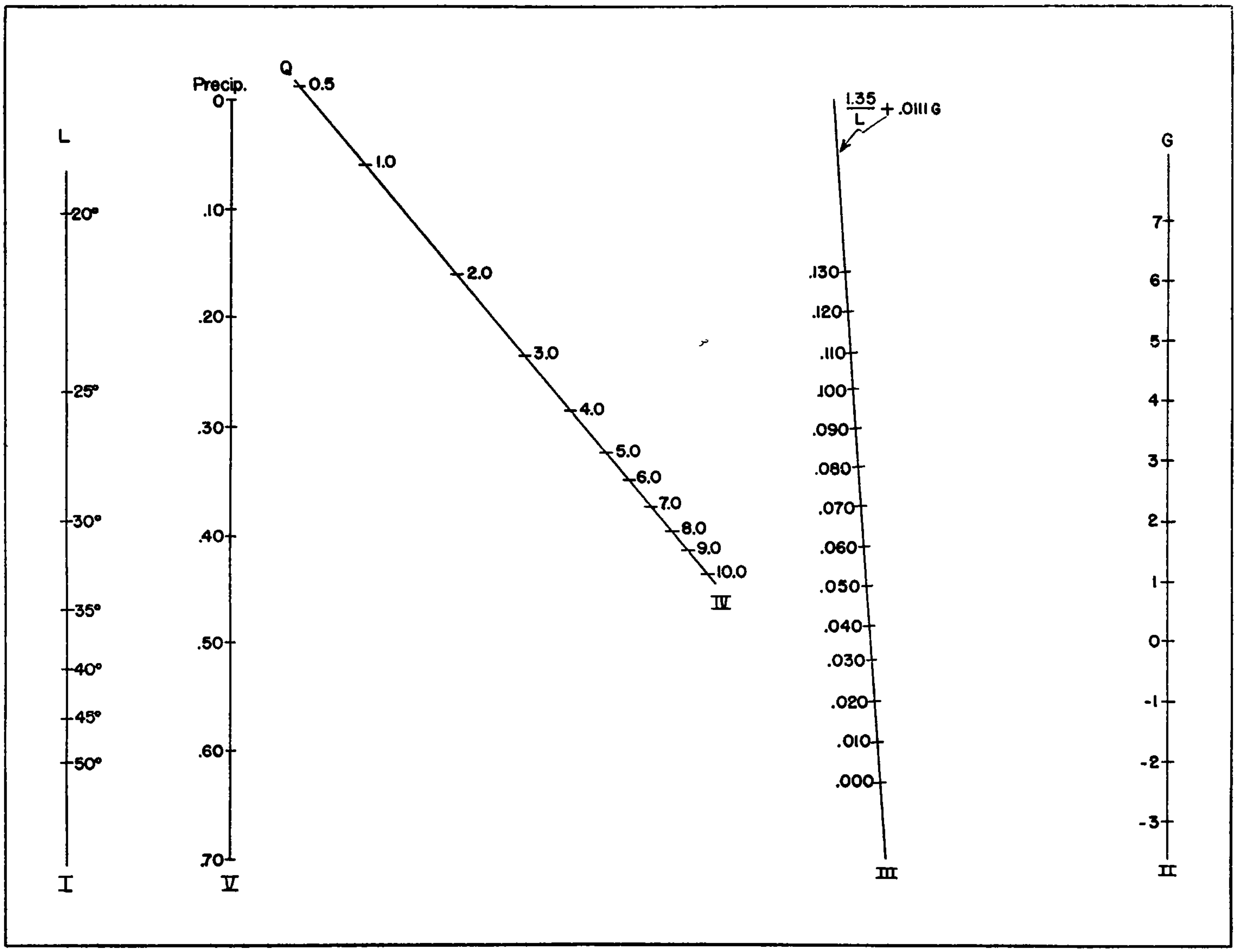

FIGURE 2.—A nomograph designed to give the solution to the equation *Precipitation*=1.35 Q/L+.0111 QG−.10

USE OF THE FORMULA

In order to simplify the computations involved in using the formula, a nomograph (fig. 2) was devised which, with two settings of a straightedge, performs all the necessary arithmetic. L and G are first combined by setting a straightedge at the appropriate points on lines I and II. The resultant combination is found on line III. Using this point and the value of Q on line IV the precipitation amount is read from line V.

In general the forecaster should try subjectively to incorporate other factors into the method. If, for example, the low pressure center associated with the rain area is forecast to pass well to the south of Washington, it would indicate that L and G are not giving good indications of the vertical velocity and the forecast could be reduced accordingly. Perhaps the 850-mb. level is very dry but is just above a moist layer; this fact may be used subjectively to increase Q. Subjective changes should always be considered in terms of modifications of the variables in the formula.

A study of some of the large errors indicated that there is some difficulty with those cases wherein the computed amount was very large and little or no precipitation was observed. Investigation of these cases showed unseasonably warm temperatures and pressure patterns more typical of spring or fall than winter. Although only three such cases occurred in three years of data, these atypical factors should not be ignored in making a forecast.

CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

From the limited samples used it appears possible to derive useful forecasting methods by the application of a statistical analysis to a simple meteorological hypothesis. The hypothesis makes it possible to assess the role of each of the variables and to use them in the manner suggested by theory. By so doing it is easier for the practicing forecaster to modify each individual case to conform with his subjective analysis of the situation and also to project the method further into the future. It would be desirable to test this formula on more independent samples and at the same time search for additional variables. The patterns of surface pressure systems should contain valuable clues as to the amount of upward motion to be expected, or some measure of the moisture at lower levels might be found useful. Notes collected on the successful subjective changes may give clues to such variables. It is strongly urged that any new or different variables be incorporated into the system with due regard for their physical meaning.

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